International Revista of Physics, Chemistry, Mathematics and Biology
(ISSN: xxxx-xxxx)
Vol1, Issue 1, (2026)
https://irpcmb.com/

Fractional Models for Biological Population Dynamics

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Citation: Dr. Priya Nair (2026) Fractional Models for Biological Population Dynamics, *International Revista of Physics, Chemistry, Mathematics and Biology, Vol. 1: Issue 1 (2026).*

Received: Feb 06, 2026; **Accepted**: Feb 15, 2026; **Published**: June 28, 2026

Abstract

Population models often assume that the instantaneous rate of change depends only on the present population. Biological systems can also retain the effects of earlier crowding, resource depletion, maturation and environmental exposure. Fractional differential equations represent this history through nonlocal derivatives. This review explains the Caputo fractional derivative, examines fractional logistic and predator–prey systems, and compares their interpretation with ordinary differential equations. An illustrative numerical experiment varies the fractional order in a logistic model while holding all other parameters fixed. The resulting trajectories show how the chosen memory kernel changes the timing of growth, although this demonstration is neither fitted to observations nor evidence that a fractional model always predicts better. The article closes with practical guidance on units, parameter identification, numerical validation and biological interpretation. The literature supports fractional models as a useful extension when memory is plausible, provided their extra flexibility is tested against independent data [1–4].

Keywords: fractional calculus; Caputo derivative; logistic growth; population dynamics; predator–prey interaction; memory effect; numerical simulation

1 Introduction

The logistic equation is a compact description of growth under limited resources. With population N(t), intrinsic rate r and carrying capacity K, it reads $dN/dt = rN(1 - N/K)$. Its instantaneous growth rate is determined by the current value of N. This Markovian simplification is useful, but past conditions can continue to influence fertility, survival, habitat quality and the age structure of a population. Delays and stage-structured equations are direct ways to represent such mechanisms; fractional derivatives offer a different, distributed-memory representation.

Fractional models are particularly interesting when many past time points exert progressively changing influence. Published studies have applied fractional logistic models to demographic trajectories and fractional predator–prey models to ecological interactions [2–4]. Their biological usefulness still depends on whether the memory term captures a plausible mechanism and improves out-of-sample performance.

2 Fractional calculus and biological memory

For $0 < \alpha < 1$, the Caputo derivative is $cD^{\alpha}_{t}N(t) = [1/\Gamma(1 - \alpha)] \int_0^t N'(s)/(t - s)^{\alpha}\, ds$. Γ is the gamma function and α is the fractional order. The integral combines earlier rates of change with a power-law kernel: recent history receives more weight, while older history can remain relevant. At $\alpha = 1$ the corresponding model reduces to an ordinary first derivative [1]. A fractional derivative by itself does not prove the existence of biological memory; it specifies one testable form for it.

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The standard Caputo logistic model is $cD^{\alpha}_{t}N(t) = r^{\alpha} N(t)[1 - N(t)/K]$, $N(0) = N_0$, $0 < \alpha \leq 1$. Here r^{α} denotes a coefficient with units $\text{time}^{-\alpha}$, rather than the ordinary rate raised mechanically to a power. Writing the coefficient as λ^{α} or simply λ_a can avoid confusion. Its equivalent Volterra equation is $N(t) = N_0 + [r^{\alpha}/\Gamma(\alpha)] \int_0^t (t - s)^{\alpha-1} N(s)[1 - N(s)/K]\, ds$. This form makes the dependence on the trajectory explicit [1].

The equilibria follow from the right-hand side: $N^* = 0$ and $N^* = K$. Their locations match the ordinary logistic model, but the path and timescale of approach can differ. An order parameter may absorb omitted structure, changing growth rates or noise; it should not automatically be interpreted as a measured biological memory duration.

3 Model families

Table 1. Conceptual model comparison. x denotes prey, y predator, and τ a fixed delay. The last two rows are model templates, not fitted equations.

Model	Representative equation	Biological question
Classical logistic	$dN/dt = rN(1 - N/K)$	How does density limit one population?
Fractional logistic	$cD^{\alpha}N = \lambda_a N(1 - N/K)$	Does distributed history alter the growth path?
Fractional predator–prey	$cD^{\alpha}x = ax - bxy$; $cD^{\alpha}y = cxy - dy$	How does memory affect coupled species?
Fractional with explicit delay	$cD^{\alpha}x(t) = F[x(t), y(t - \tau)]$	Do distributed memory and a fixed lag both matter?

4 Predator–prey interactions and stability

A fractional Lotka–Volterra system replaces first derivatives with fractional ones: $cD^{\alpha}_{t}x = ax - bxy$ and $cD^{\alpha}_{t}y = cxy - dy$, with positive coefficients a, b, c and d. Births increase prey, encounters remove prey, conversion supports predators and mortality decreases predators. Extensions include saturation of predation, harvesting, spatial movement and explicit time delays. Li and colleagues investigated a fractional delayed predator–prey model with feedback and analysed how delay and fractional order affect bifurcation behavior [3]. Their findings apply to the specified model and parameter regimes, not to every ecosystem.

Equilibrium analysis remains a first check. The positive equilibrium of this simple system is $(x^*, y^*) = (d/c, a/b)$. The Jacobian and the fractional order jointly influence local stability. For an autonomous commensurate Caputo system with $0 < \alpha < 1$, a common linearized stability criterion requires every Jacobian eigenvalue λ to satisfy $|\arg \lambda| > \alpha\pi/2$. This angular condition differs from the classical left-half-plane test and should be applied with attention to model assumptions [1].

Ecologically, the order α can change transient overshoot or damping even when a formal equilibrium is unchanged. Researchers should distinguish an observed generational lag from long-tail distributed memory. A fixed maturation period might be represented more transparently by a delay differential equation; both approaches can be compared empirically rather than selected by convenience.

5 Numerical method for the illustrative figure

The figure uses a product-integration approximation of the Caputo integral equation for the logistic model. At step n and step size h, $N_n = N_0 + [\lambda_a h^{\alpha}/\Gamma(\alpha + 1)] \sum_{j=0}^{n-1} [(n - j)^{\alpha} - (n - j - 1)^{\alpha}] N_j(1 - N_j/K)$. The resulting explicit history sum uses all earlier states. It is included for reproducibility and illustration; a converged predictor–corrector scheme is preferable for quantitative inference. We used $h = 0.05$, $N_0 = 10$, $K = 100$, $\lambda_a = 0.35$ in the corresponding $\text{time}^{-\alpha}$ units, and a horizon of 12 time units.

6Illustrative results and discussion

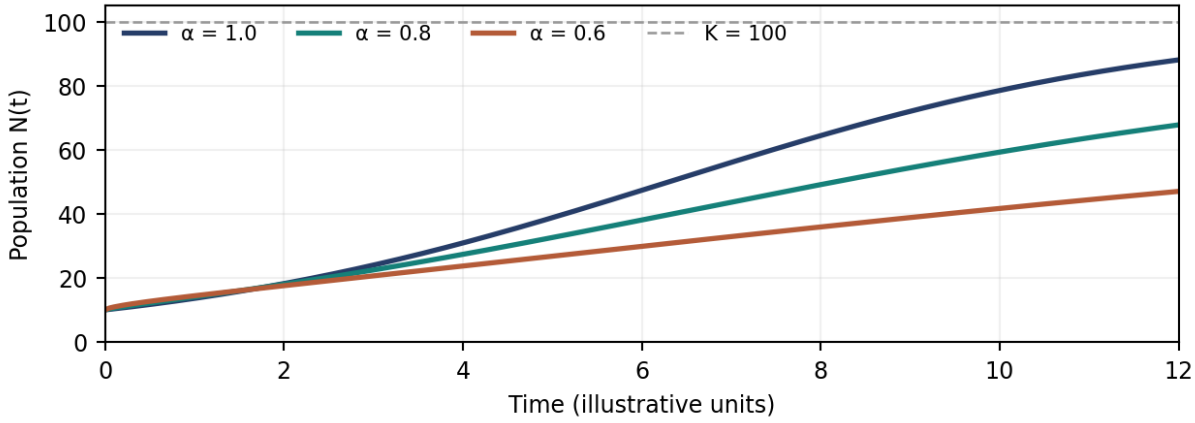


Figure 1. Simulated fractional logistic trajectories with α = 1.0, 0.8 and 0.6. These are constructed model outputs, not observed biological or census data. The same numerical coefficient is used across orders solely to show sensitivity; its physical units change with α.

Table 2. Simulated population values from the discretization stated in Section 5; rounding is to one decimal place.

Time	α = 1.0	α = 0.8	α = 0.6
0	10.0	10.0	10.0
3	24.0	22.5	20.6
6	47.3	38.1	29.8
9	72.0	54.4	38.8
12	88.1	67.8	47.0

The trajectories differ because each α specifies a different integration kernel and a differently dimensioned coefficient. Consequently, a direct visual ranking of "fastest" biological growth is not meaningful without calibrating comparable timescales and uncertainty. The values show sensitivity under one parameter choice, not universal monotonic behavior in α. A credible empirical comparison would fit each model on the same training period, preserve the same observation process, then compare held-out error and uncertainty intervals.

A published Bangladesh population analysis applied a non-singular fractional logistic formulation and multi-scale approximations to census series [2]. That study motivates empirical investigation, but its Atangana–Baleanu–Caputo operator is distinct from the singular-kernel Caputo model plotted here. Comparing their α values directly would therefore be inappropriate. Similarly, an Allee effect, seasonal forcing or changing K may explain trajectories otherwise attributed to memory [4].

7 Practical limitations and conclusion

Fractional models add a history term and an order parameter. This can represent persistent influences, but also creates identifiability problems: growth rate, carrying capacity, initial conditions and α may trade off against one another. Fractional derivatives can embody different kernels and initial-condition conventions. Researchers should name the operator, state parameter units, report solver and step-size convergence, and perform sensitivity analysis. Positivity of populations and the admissible domain of the solution should be checked explicitly. Measurements spanning too few generations may be unable to distinguish power-law memory from a small number of ordinary delayed or stage-structured processes.

In conclusion, fractional logistic and predator–prey equations supply a flexible language for distributed history in biological populations. The illustrative calculations demonstrate that changing α can materially alter transients while leaving the logistic carrying capacity fixed. A

claim of biological memory requires mechanistic reasoning, appropriate data and validation against simpler alternatives. The fractional order is most useful as an estimable and falsifiable model component, rather than as an automatic improvement over classical equations.

References

[1] Diethelm, K. (2010). The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type. Springer, Lecture Notes in Mathematics 2004. https://doi.org/10.1007/978-3-642-14574-2

[2] A non-singular fractional-order logistic growth model with multi-scaling effects to analyze and forecast population growth in Bangladesh. (2023). Scientific Reports, 13, 20118. https://doi.org/10.1038/s41598-023-45773-1

[3] Li, Z., Zhang, W., Huang, C., & Zhou, J. (2021). Bifurcation for a fractional-order Lotka–Volterra predator–prey model with delay feedback control. AIMS Mathematics, 6(1), 675–687. https://doi.org/10.3934/math.2021040

[4] On the fractional Allee logistic equation in the Caputo sense. (2023). Partial Differential Equations in Applied Mathematics. https://www.sciencedirect.com/science/article/pii/S2666657X2300023X

Article type: Narrative review and transparent illustrative simulation. No new empirical dataset or original field observations are claimed.